RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. SECOND SEMESTER (January – June) 2014

Mid-Semester Examination, March 2014

Date : 27/03/2014 **MATHEMATICS (General)**

Time : 11 am – 12 noon Paper : II Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

1. Answer **any five** questions : [5×1]

   a) What will be the equation of $x^2 + y^2 = 1$ if origin (0,0) is shifted to the point (1,0)?

   b) Define rotation of an orthogonal system of coordinate axes?

   c) Find the equation of bisectors of the pair of straight lines $2x^2 + 3xy + 4y^2 = 0$.

   d) What will be the value of θ by which the orthogonal system through (0,0) rotated affects xy = 1 to becomes xy = –1.

   e) Find the value of 'a' when y = ax is a bisector of $x^2 + xy + y^2 = 0$.

   f) Find the angle between the pair of straight lines represented by the equation $x^2 + 5xy + 3y^2 = 0$.

   g) When it is possible that $ax^2 + 2hxy + by^2 + 2gx + 2fy = 0$ represents a pair of straight lines does not passing through origin.

2. Answer **any one** question : [1×5]

   a) i) Define collinear vectors.

   ii) Show that the points $A(\vec{i} - 2\vec{j} + 3\vec{k})$, $B(2\vec{i} - 3\vec{j} + 4\vec{k})$ and $C(-2\vec{i} + \vec{j})$ are collinear. [1+4]

   b) i) If $|\vec{\alpha} + \vec{\beta}| = |\vec{\alpha} - \vec{\beta}|$, find the angle between $\vec{\alpha}$ and $\vec{\beta}$.

   ii) Show by vector method, that the perpendicular from the vertices of a triangle to the opposite sides are concurrent. [2+3]

3. Answer **any one** question : [1×5]

   a) Define a bounded sequence. Show that every convergent sequence is bounded. [2+3]

   b) Show that the sequence $\left\{\frac{4n+3}{3n+4}\right\}$ is convergent. Find the limit of the sequence. [4+1]

Group – B

4. Answer **any one** : [1×5]

   a) Find the value of $\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \ldots + \frac{1}{n}\right]$.

   b) If $I_n = \int_0^{\pi/4} \tan^n x\,dx$ where n is a positive integer, show that $I_n = \frac{1}{n-1} - I_{n-2}$. Use this result to evaluate $\int_0^{\pi/4} \tan^6 x\,dx$.

5. Answer **any one** of the following : [1×5]

   a) Find the Integrating factor of the following differential equation and hence solve : $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$.

   b) Solve the differential equation : $y + px = x^4p^2$. Also determine the Singular solution.